Two-Way ANOVA

When running a two-way ANOVA we have to check the procedure's formal assumptions, which are identical to those of its one-way counterpart. While most assumptions are easily testable, SPSS unfortunately does not provide us with an alternative test - such as the Welch's test - for situations in which we have to reject the Levene's test of homogeneity of variance. If Levene's test suggests unequal population variances, you can routinely interpret the analysis results (i.e. the different F-tests for the main and interaction effects) but assume a more stringent significance level such as 0.01. Subsequently, you should consider the main and interaction effects, which are only significant if the p-values are smaller than 0.01 (instead of the commonly used threshold value of 0.05).

As discussed in Chapter 6 of the book, we need to consider several different types of variations when running a two-way ANOVA: (1) the total variation (SS_T), (2) the between group variation in factor 1 (i.e. promotion campaigns; SSB1), and (3) the between-group variation in factor 2 (i.e. service type; SS_{B2}), (4) the variation due to the interaction of factors 1 and 2 (SS_{B1x2}), and (5) the within-group variation (SS_W).

With these different types of variations, there are three null hypotheses that are of particular interest to us:

1) The factor level means of the first factor are equal in the population
2) The factor level means of the second factor are equal in the population, and
3) There is no interaction effect between the two factors.

Note that in theory, we could test further hypotheses (e.g., the population means of the first or second factor are equal) which, however, have little practical use. Before testing any of these hypotheses, we have to make sure that the assumptions described above hold. That is, we have to test whether the data are normally distributed, and whether the population variances are equal. Furthermore, this step involves ensuring that the samples are independent and that the sample sizes are similar in each group. Once this is done, we can proceed by decomposing the total variation. As you will see, we are very fortunate that SPSS does this job for us since the numerous indices can be quite confusing. Nevertheless, it's worthwhile taking a look at the formulae. You will see that the basic concept underlying the two-way ANOVA is essentially the same as with the one-way ANOVA.

Let's take a closer look at the different types of variation that need to be computed. We already calculated the overall variation ($SS_T = 584$) as well as the between-group variation for the first factor ($SS_{B1} = 273.80$) so that we still need to consider the following three types of variation:

1) The *between-group variation* in factor 2 (i.e. service type), computed by comparing each group's mean sales $\bar{x}_1$ (i.e. the sales means for personal as well as self service) with the overall mean $\bar{x}$, weighted by n_l, the number of observations in the group (i.e. 15 in both groups). Overall, there are m factor levels (groups) in factor 2. For this purpose, we first need to calculate the mean values for the two service types, personal service ($\bar{x}_1 = 49.27$) and self service ($\bar{x}_2 = 46.73$). Thus, we can compute the following:

$$\begin{aligned} SS_{B2} &= \sum\nolimits_{l=1}^{m} n_1 (\bar{x}_l - \bar{x})^2 \\ &= 15 \cdot [(50 - 49.27)^2 + \cdots + (47 - 49.27)^2] + 15 \cdot [(45 - 46.73)^2 \\ &\quad + \cdots + (44 - 46.73)^2] \\ &= 48.13. \end{aligned}$$

2) To calculate the *variation due to the interaction* between the two factors, we need to consider yet another mean value $\bar{x}_{jl}$, which describes the mean sales in each of the six factor level combinations (i.e. the mean sales for personal service combined with point of sale display ($\bar{x}_{11} = 48$) all the way through the mean sales for self service combined with in-store announcements ($\bar{x}_{33} = 48.30$)). n_{jl} describes the number of observations in each of the factor level combinations which, in our example, is always 5.

$$\begin{aligned} SS_{B1x2} &= \sum\nolimits_{j=1}^{k} \sum\nolimits_{l=1}^{m} \mathrm{n_{jl}} \cdot (\overline{\mathrm{x}}_{\mathrm{jl}} - \overline{\mathrm{x}}_{\mathrm{j}} - \overline{\mathrm{x}}_1 + \overline{\mathrm{x}}) \\ &= 5 \cdot (48 - 47.3 - 49.27 + 48) + 5 \cdot (46.60 - 47.3 - 46.73 + 48) \\ &\quad + 5 \cdot (54.20 - 52 - 49.27 + 48) + 5 \cdot (49.80 - 52 - 46.73 + 48) \\ &\quad + 5 \cdot (45.60 - 44.7 - 49.27 + 48) + 5 \cdot (43.80 - 44.7 - 46.73 + 48) \\ &= 13.27 \end{aligned}$$

3) The *within group variation* SSW. Since we have a second factor in our model, of course, the overall within-group variation changes compared to that of the one-way ANOVA. Specifically, it is computed as follows:

$$\begin{aligned} SS_W &= \sum\nolimits_{j=1}^{k} \sum\nolimits_{l=1}^{m} \sum\nolimits_{i=1}^{n} (x_{ijl} - \bar{x}_{jl})^2 \\ &= (50 - 48)^2 + (52 - 48)^2 + \cdots + (47 - 43.80)^2 + (42 - 43.80)^2 \\ &= 248.80 \end{aligned}$$

As in the one-way ANOVA, we convert these estimates into mean squares by dividing each by its degrees of freedom, which yields the following:

– Mean square between-group variation factor 1 (promotion campaign):

$$MS_{B1} = \frac{SS_{B1}}{k-1} = \frac{273.80}{3-1} = 163.90$$

– Mean square between-group variation factor 2 (service type):

$$MS_{B2} = \frac{SS_{B2}}{m-1} = \frac{48.13}{2-1} = 48.13$$

– Mean square interaction effect:

$$MS_{B1x2} = \frac{SS_{B1x2}}{(k-1)\cdot(m-1)} = \frac{13.27}{2\cdot 1} = 6.63$$

– Mean square within-group variation:

$$MS_W = \frac{SS_W}{n-k\cdot m} = \frac{248.80}{30-6} = 10.37$$

We can now use these estimates to test the different effects described above. As in a one-way ANOVA, we divide the mean square values to test the desired effects: To test whether the main effect of factor 1 is significant, we calculate $F = \frac{MS_{B1}}{MS_W} = \frac{163.90}{10.37} = 13.21$, which follows an F-distribution with *(k–1)* and *(n–k·m)* degrees of freedom (i.e. 2 and 24 in our example). As the resulting critical value of 3.40 (for a significance level of 5%) is clearly smaller than the test value of 13.21, we can reject the null hypothesis and conclude that the promotion campaign factor exerts a significant influence on sales.

For the second factor, we get $F = \frac{MS_{B2}}{MS_W} = \frac{48.13}{10.37} = 4.64$, which also follows an F-distribution but with *(m–1)* and *(n–k·m)* degrees of freedom. As in the case of factor 1, the critical value (4.26 for $\alpha = 0.05$) is smaller than the test statistic, providing support for the type of service also having a significant bearing on sales.

Finally, we test the interaction effect by computing the following test statistic: $F = \frac{MS_{B1x2}}{MS_W} = \frac{6.63}{10.37} = 0.64$, which follows an F-distribution but with *(k–1)·(m–1)* and *(n–k·m)* degrees of freedom. Whereas the two main effects of factors 1 and 2 were significant, this is clearly not the case with the interaction effect. The test value (0.64) lies far below the critical value (3.40 for $\alpha = 0.05$).

Figure A6.1 shows the results graphically. The shape of the lines shows that regardless of the type of service, sales are in both cases highest for the free tasting stand, followed by the point of sale display, and, finally by the in-store announcements. However, factors only interact if the effect of one of the factors differs depending on the level of the other factor. The fact that the lines are almost parallel across the three levels of factor 1 clarifies that there is no interaction present between the two factors. If this were the case, there would be pronounced differences in the lines' slopes (e.g., one line having a highly negative slope, with the other having a highly positive slope).

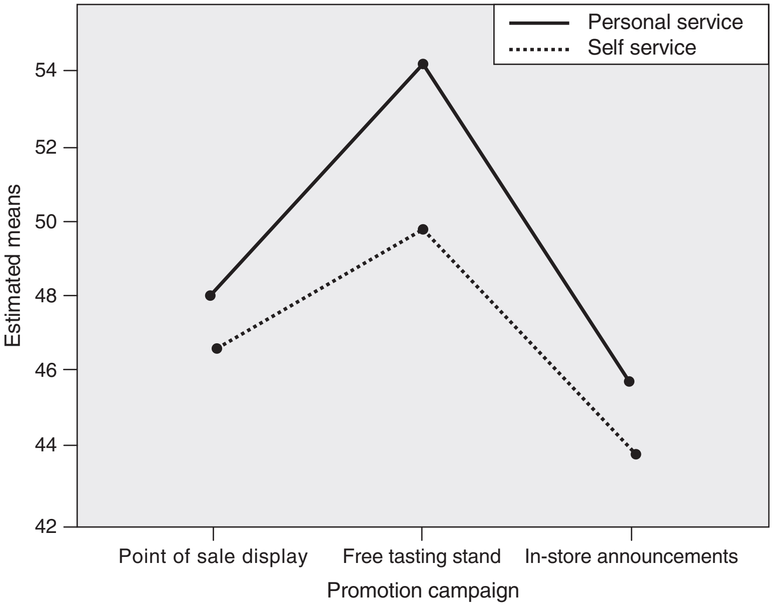

Figure A6.1 Visual inspection of interaction effects.